Stream Ciphers

Another popular type of cipher is the **stream cipher**, which operates on variable lengths of data that come in as a "stream". You can think of it like encrypting a phone call in real-time – we don't know the length of data beforehand. We could try to use a block cipher, but this can get messy since it requires data in fixed-length blocks before we send it over.

If we had a long, secure bitstring, we could just XOR the plaintext with that and send it immediately (exactly like the one-time pad). As we saw with the one-time pad, however, this is a bit infeasible. Instead, what if we use the output of a CSPRNG as the bit string? Sure, this is not ideal, but if the PRNG output is reasonably good, it will be good enough.

This is the basic idea behind many popular stream ciphers like RC4. Though, RC4 is considered broken nowadays due to the output leaking a significant amount of information about the key itself (attacks which are unfortunately out of scope for this class).

Stronger stream ciphers such as ChaCha20 are mainly in use today. They have much more complicated internals than just a basic PRNG, but the core idea is roughly the same.

Block Ciphers

Many cryptographic schemes are much easier to build when we consider fixed-length *blocks* of inputs to do operations on, rather than considering arbitrarily-long data. Block ciphers operate this way, taking in some input, splitting it up into blocks, encrypting each block, and recombining afterward. Note that the overall data can be arbitrarily long, as long as we keep each block to a fixed size (the number of blocks does not matter). We will primarily be focusing on AES (Advanced Encryption Standard), which is the premier block cipher in use today. AES is so secure that even the NSA approves its use for top-secret data. No (practical) attacks are known on correctly-implemented AES as of 2022 [1].

The specifics of the cryptography within the actual "operation" can be found in the appendix, as it is rather detailed. You can think of it like a function $f(k,d) = c$ taking in a fixed-length key k and data d of the same length, where c is the output (again of the same length). The actual operation within these blocks is considered the core "algorithm", whereas how we *use* it is the focus of modes of operation.

Block ciphers are expected to have two key properties – **confusion** and **diffusion**. Confusion is the property that each bit of the output should be influenced by many parts of the secret key, not just

[1] There is a largely theoeretical attack named the *biclique attack* that shaves a few bits off the keysize, but it is both infeasible to implement and not worth the tradeoff.

one or two bits of the key. Note that basic substitution ciphers have no confusion at all – each bit is uniquely influenced by one bit in the secret key. **Diffusion** is the property that a one-bit change in either the plaintext or key should change about half of the output bits. This works to prevent statistical analysis of the output by flipping input bits. You can visit the appendix for a detailed example of how AES accomplishes these two properties.

Modes of Operation

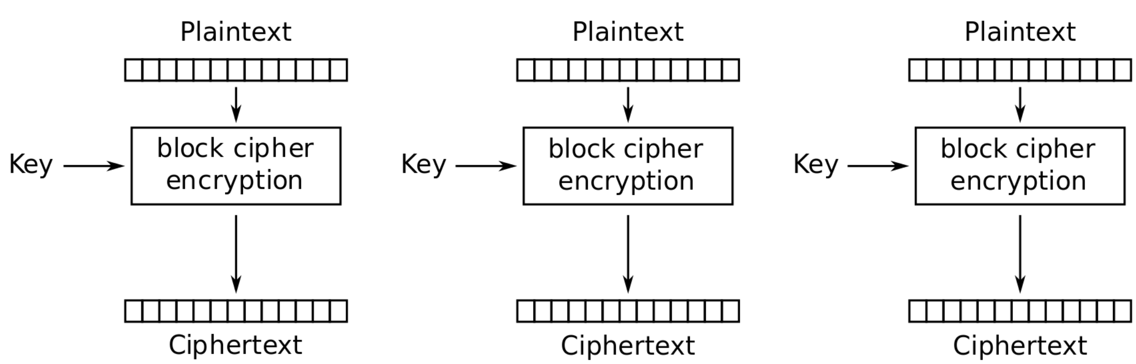
We now have our block cipher function $f(k,d)$, but how do we use it to encrypt arbitrarily long data? First, we split the data up into blocks $b_1, b_2, \ldots b_k$, padding as required. Once we have these, we must figure out how to order our encryption.

The most straightforward way of encrypting might be to set $c_i = f(k, b_i)$ for all $i = 1, \ldots, k$. In effect, we encrypt every block individually with the same key and output the ciphertext blocks respectively. Assuming that f is secure, this won't immediately *leak* our encrypted message, but it falls prey to a weaker attack. Say we were commanding some invasion force, monitoring some early-warning radar station. Every 15 minutes, the station broadcasts an encrypted message "ALL CLEAR". If they spot us, they will instead broadcast some other message. We only need to know when we've been spotted. Can you think of why we can easily do this?

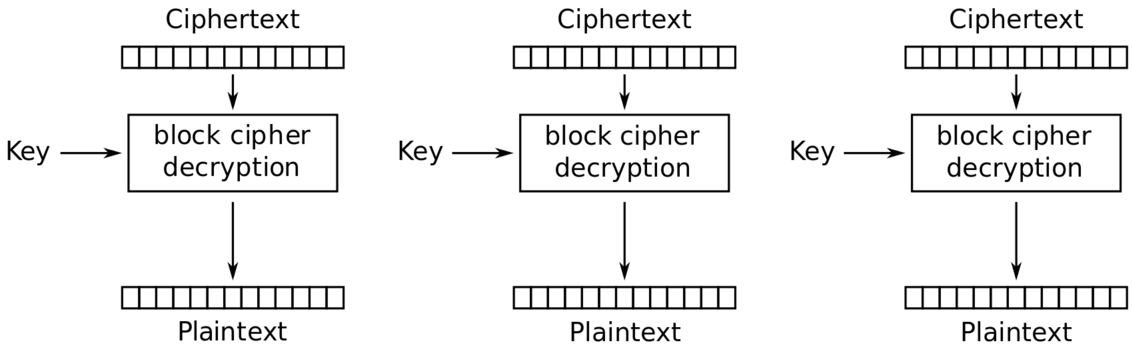
The reason is that this AES mode is entirely *deterministic*, in that encrypting the same message will always output the same ciphertext. So if we ever see a different message than has been sent before, we know we've been spotted. Even worse, we can tell exactly which blocks in the message have changed. If we only changed b_3, then only c_3 will have changed in the output!

This represents a lack of IND-CPA security. To see this formally, recall the example given for a deterministic function in the IND-CPA section of this note.

A visual aid for how ECB works (courtesy of CS 161 textbook):



Electronic Codebook (ECB) mode encryption



Electronic Codebook (ECB) mode decryption

AES-CBC, or Cipher Block Chaining, is a secure (and widely used) mode of operation for AES. Instead of blindly applying the key linearly, it utilizes the result of the previous encryption to act as a source of "randomness" for the next one (note it isn't actually random). At any given step i, we have to know C_{i-1}, P_i, and k to compute C_i. The formula is:

$$C_i = \text{Enc}(k, C_{i-1} \oplus P_i)$$

We can reverse this to find the decryption formula:

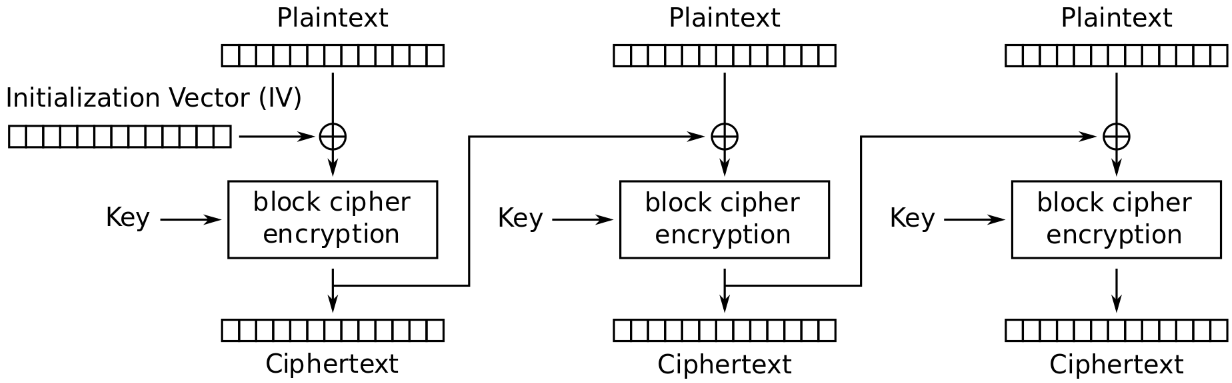
$$P_i = \text{Dec}(k, C_i) \oplus C_{i-1}$$

Informally, we encrypt the XOR of the previous ciphertext block and the current plaintext block with the key. This means any difference in the previous block would "carry forward" and change the results of the future blocks. Remember that the output of a good block cipher is (practically) indistinguishable from random noise, so even a small random change in block one would be enough to make the entire ciphertext look totally different.

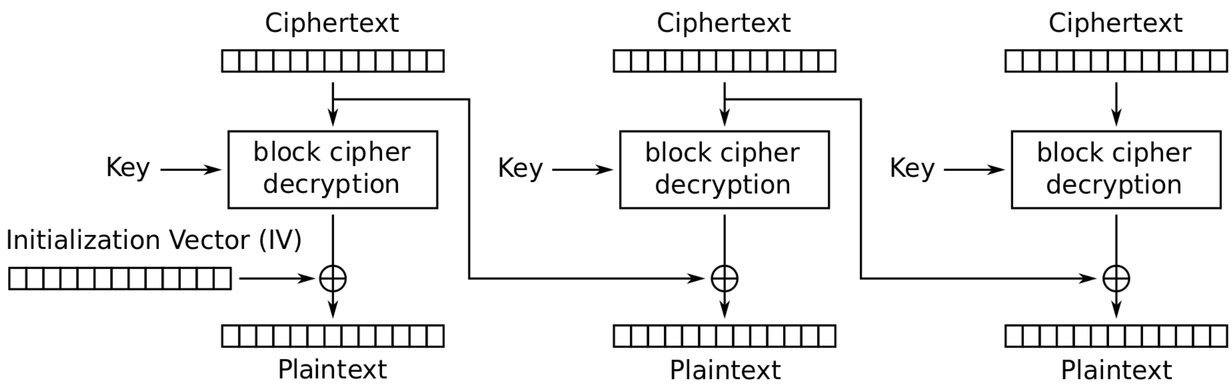
This raises the question – how do we encrypt C_1? We would need the previous ciphertext, but that doesn't exist. Enter the concept of the **initialization vector**, a fancy term for a "ciphertext" block that we provide to start the encryption. **In order for AES-CBC to be secure, this IV must be randomly generated each time we send a new message!** Can you see why AES-CBC might not be IND-CPA secure if we re-use the IV?

Note that the receiver must also know the IV to decrypt the message. For this reason, IVs are considered to be public knowledge, since we send $[IV, C_1, C_2, \ldots C_k]$ as the message. This does not affect security, however (assuming you didn't reuse the IV)!

Here is a visual aid of AES-CBC, again courtesy of the CS 161 textbook:



Cipher Block Chaining (CBC) mode encryption



Cipher Block Chaining (CBC) mode decryption

It turns out we can decrypt every block at once since we know the entirety of C, but encrypting is a painfully sequential process.

Constructing Block Ciphers

The construction of the actual encryption function is usually done via a **substitution-permutation network** (SPN), which involves the chaining of substitution boxes and permutation boxes. A substitution box involves mapping all 4-bit inputs (for example) to unrelated 4-bit outputs. This provides **confusion**, a desirable property of ciphers which states the output should rely on all parts of the input. A permutation box maps the location of the bits to new locations, achieving **diffusion** (the property that one bit change in the input should result in about 50% of the bits changing).

After a substitution and permutation, the internal state is usually XOR-ed with a **round key**. These are derived from the original key via the **key schedule**.

Linear Cryptanalysis

Some poorly-constructed block ciphers are vulnerable to **linear cryptanalysis**. This techniques uses the linearity of XOR to exploit sboxes. We want to find an equation of the form

$$P_1 \oplus P_2 \ldots C_1 \oplus C_2 \ldots = K_1 \oplus K_2 \ldots$$

(with various entries, not all will have K_1, etc). In an ideal cipher, every equation of this form will hold with exactly 0.5 probability. If we can find one of these **linear approximations** that holds with greater probability, we can exploit to learn the round keys.

For example, consider the linear approximation of

$$P_1 \oplus P_4 \oplus C_2 \oplus C_4 = K_1 \oplus K_2$$

that holds with 70% probability. We can generate some large enough number of plaintext-ciphertext pairs and evaluate this equation for each of them, having guessed the values of K_1 and K_2 (guessing specific bits is possible since the round key schedule is public and deterministic). We can test all 4 possibilities for the combinations of K_1 and K_2 and pick the one that occurs the most frequent – we expect the right choice of K_1 and K_2 to satisfy the equation with 0.7 probability.

Differential Cryptanalysis

Another technique for block-cipher cryptanalysis is that of **differential cryptanalysis**. Instead of linear equations, we focus on **differentials**, which are specific XOR differences between an input and output of a substitution box:

$$\Delta_y = SBOX(X \oplus \Delta_x) \oplus SBOX(X)$$

For example, the pair (5, 7) means if two inputs differ by an XOR of 5, their outputs will differ by an XOR of 7. In a one-sbox cipher, we can narrow down the possibilities for the round keys by finding a **good pair**, a pair of plaintext/ciphertext pairs that exhibits this desired differential. We then consult a lookup table of what inputs to the SBOX allow for such a differential, and conclude that the internal sbox input (post-round 1 XOR) must be one of these inputs. From there, we can brute force the possible inputs and derive the respective round keys using the plaintext/ciphertext pair.

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Appendix: AES Internals

Coming soon!